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Investigation of Temperature-Dependent Parameters Effect on Thermal Fin Performance Using the Response Surface Methodology

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Abstract

Thermal fins, as one of the key components in enhancing the performance of heat transfer systems, play a vital role in various industries. This research investigated the effect of temperature-dependent parameters such as thermal conductivity, heat transfer coefficient, and fin geometry on thermal efficiency. For this purpose, the response surface methodology was used. The results showed that the proposed model, with a coefficient of determination (R^2) of 0.990 and a signal-to-noise ratio of 180.94, is very accurate in predicting the system behavior. The analyses showed that increasing the geometric parameter M, as the ratio of convective heat transfer to thermal conductivity, as well as the geometric effect of the fin, leads to a decrease in thermal efficiency because an increase in the convective heat transfer rate intensifies the concentration of temperature in parts of the fin. The parameter n, which determines the degree of dependence of the heat transfer coefficient on temperature, reduces the overall efficiency by concentrating the heat transfer rate in the hotter areas of the fin. In contrast, the parameter β, which indicates the dependence of thermal conductivity on temperature, has less influence on the overall performance of the fin but can help to make the temperature distribution more uniform. The use of the response surface method not only reduces the computational cost, but also allows for rapid and accurate analysis of complex conditions. The results of this research can be used as an effective guide for the optimal design of fins in the heat transfer industries.

Keywords

Thermal fin; Response surface methodology; Thermal conductivity; Heat transfer coefficient; Temperature dependency; Fin efficiency

1. Introduction

Heat transfer is a fundamental phenomenon in engineering, with fins widely used to increase surface area and improve heat dissipation in systems such as air conditioners, aircraft engines, and computer processors [1-3]. Analyzing fin thermal behavior requires a deep understanding of heat transfer dynamics. While assuming constant thermal conductivity (K) and heat transfer coefficient (h) allows for analytical solutions, real-world applications with large temperature gradients render these parameters temperature-dependent, leading to nonlinear governing equations [4]. Numerical methods (e.g., FEM, FDM) and semi-analytical methods (e.g., HAM, DTM) have been developed to solve these nonlinear problems [5, 6]. However, these methods can be computationally intensive.

Response Surface Methodology (RSM) is a powerful statistical technique for modeling and optimizing complex processes by examining the effects of multiple variables and their interactions [7]. This study employs RSM to analyze the effects of key parameters (thermo-geometric parameter (M), temperature dependency of the heat transfer coefficient (n), and temperature dependency of thermal conductivity (β)) on fin efficiency (η). The novelty lies in using RSM to derive a simple, explicit polynomial model from detailed DTM simulations, enabling rapid sensitivity analysis and providing practical design guidelines. This approach reduces computational cost while maintaining high predictive accuracy.

2. Methodology

The problem involves a one-dimensional straight fin with constant cross-sectional area (A_c), perimeter (P), and length (L), attached to a base at temperature T_b and exposed to a fluid at T_a with an insulated tip. The steady-state energy equation is given by (1):

$$A_C \frac{d}{dX}\left(K(T)\frac{dT}{dX} \right) - P\,h(T)(T-T_a) = 0, \qquad 0 < X < L \tag{1}$$

The temperature-dependent heat transfer coefficient and thermal conductivity are defined as power-law functions:

$$h = h(T) = h_b \left(\frac{T-T_a}{T_b-T_a} \right)^n \tag{2}$$

$$K = K(T) = K_b \left(\frac{T-T_a}{T_b-T_a} \right)^\beta \tag{3}$$

After non-dimensionalization, the governing equation becomes (4):

$$\frac{d}{dx}\left[\theta^\beta \frac{d\theta}{dx} \right] - M^2\theta^{n+1} = 0, \; 0 \le x \le 1 \tag{4}$$

where $x = X/L$, $\theta = T - T_a / T_b - T_a$ and the thermo-geometric parameter $M^2 = h_b PL^2 / K_b A$ .The dimensionless boundary conditions are: at X=0 (fin tip, adiabatic) dθ/dX=0 and at X=1 (fin base) θ=1.

Fin efficiency, the ratio of actual heat transfer to ideal heat transfer (if entire fin were at base temperature), is defined as:

$$\eta = \frac{Q_{Actual}}{Q_{Ideal}} = \frac{Q = \int_0^L Ph(T)(T-T_a)\,dX}{PLh_b(T_b-T_a)} = \int_0^1 \theta^{n+1}\,dx \tag{5}$$

Given the complexity of solving (4) for many parameter combinations using the Differential Transformation Method (DTM) [8], this study uses RSM. A full factorial design with three independent variables (M, n, β), each at 6 levels (total 216 runs), was employed. The parameter ranges were: M = 0 to 1.25, n = -1.4 to 3, and β = -0.5 to +0.5, based on literature [4, 6, 8]. The Design-Expert 13 software was used for regression analysis and plotting. A quadratic polynomial model was fitted to the DTM-predicted fin efficiency data.

3. Discussion and Results

The ANOVA results for the quadratic model are summarized in Table 1. The model is highly significant ($p < 0.0001$). The coefficients of determination are excellent: $R^2 = 0.9904$ and Adjusted-$R^2 = 0.99$, indicating the model explains 99% of the variability. The adequate precision (signal-to-noise ratio) is 180.95, far above the desirable value of 4, confirming the model's high predictive capability. The final quadratic model in terms of coded factors is:

$$\begin{aligned}\eta_{Pr} &= 1.01573 - 0.145272\,M - 0.0258217\,n + 0.00390005\,\beta - 0.0683684\,Mn - 0.0239361\,M\beta + 0.000465353\,n\beta \\ &\quad - 0.0993263\,M^2 + 0.0076526\,n^2 + 0.0027609\,\beta^2\end{aligned} \tag{6}$$

Table 1- ANOVA results for the quadratic model

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	5.44	9	0.6049	2363.24	< 0.0001
A - M	4.5	1	4.5	17566.12	< 0.0001
B - n	0.6652	1	0.6652	2598.74	< 0.0001
C - β	0.0028	1	0.0028	11.07	0.001
Residual	0.0527	206	0.0003		
Cor Total	5.5	215			
R² = 0.9904,	Adj. R² = 0.99,	Adeq Precision=180.9492			

The perturbation plot (Figure 1) shows that fin efficiency is most sensitive to M (steepest slope), followed by n, while β has the least effect.

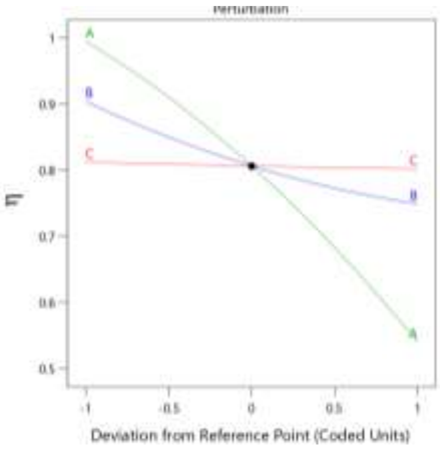

Figure 1. Perturbation plot showing the effect of *M*, *n*, and *β* on fin efficiency (*η*)

The effect of *M* (Figure 2) shows that increasing *M* decreases η. A higher *M* means increased convective heat transfer relative to conduction. This intensifies the temperature drop along the fin, making a large portion of the fin near ambient temperature and thus ineffective for heat transfer.

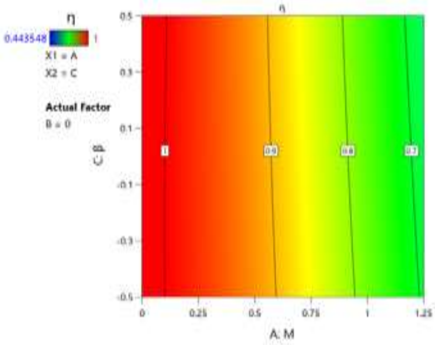

Figure 2. Effect of thermo-geometric parameter *M* on fin efficiency (*η*)

The effect of *n* (Figure 3) shows that increasing *n* decreases η. A larger *n* concentrates the heat transfer coefficient *h(T)* near the hot base, where θ is high. This reduces the contribution of the cooler distal parts of the fin, leading to inefficient use of the fin surface and lower overall efficiency.

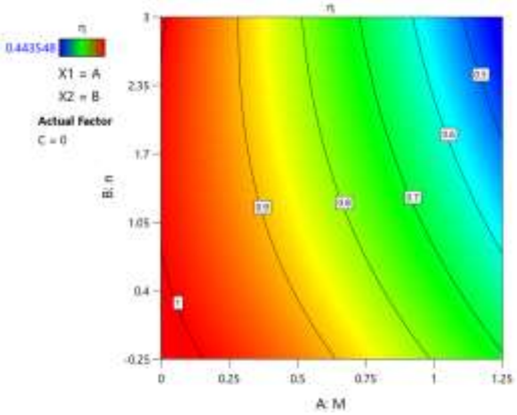

Figure 3. Effect of parameter n on fin efficiency (*η*)

The effect of β (Figure 4) shows a relatively minor influence on η compared to *M* and *n*. While β affects thermal conductivity distribution, its impact is more uniform and indirect, making it a secondary parameter in fin design optimization.

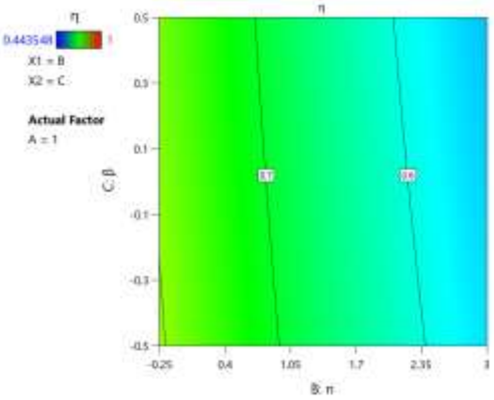

Figure 4. Effect of parameter *β* on fin efficiency (*η*)

4. Conclusions

This study successfully employed Response Surface Methodology to analyze the thermal performance of fins with temperature-dependent properties. The key conclusions are:

1. The developed quadratic RSM model ($R^2 = 0.9904$) accurately predicts fin efficiency with significantly reduced computational effort compared to numerical or semi-analytical methods.
2. The thermo-geometric parameter M has the most significant impact; increasing M sharply reduces fin efficiency by creating steep temperature gradients.
3. The temperature-dependency exponent for the heat transfer coefficient, n, also strongly affects efficiency; higher n values concentrate heat transfer near the base, reducing overall fin utilization.
4. The temperature-dependency exponent for thermal conductivity, β, has a minor effect on overall fin efficiency.
5. For optimal fin design, primary focus should be on minimizing M and n within operational constraints. RSM provides a powerful, efficient tool for such multi-parameter optimization in complex heat transfer systems.

5. References

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